Modeling loan loss provisions under IFRS 9 in the top-down solvency stress test of the Central Bank of Hungary

Draft paper
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July 9, 2019

Abstract

Stress testing frameworks are faced by challenges due to the introduction of IFRS 9, which is expected to fundamentally change banks' loan loss provisioning. To tackle this problem, we developed a credit risk stress testing model that makes it possible to forecast loan loss provisions taking into account the new expectation-based impairment requirements. We employed this framework in stress testing the corporate portfolio of the Hungarian banking sector. Our results indicate that the change in expectations due to an adverse shock has an immediate and sizable impact on loan loss provisions in contrast to the previous incurred loss approach. This might exacerbate the procyclical behavior of the banking sector.[†]

1 Introduction

Central banks' top-down stress testing practices are faced by challenges due to the changing regulatory and macroeconomic landscape. As of 2018, financial institutions preparing their financial statements according to the International Financial Reporting Standards (IFRS) must implement the new IFRS 9 standard, which deals with accounting principles for financial instruments. The new standard represents a major change in banks' loan loss provisioning practices, as it requires them to recognize impairments based on *expected losses* instead of the previously common incurred loss approach (applied in IAS 39 and in most national GAAPs). Additionally, contracts have to be classified into stages according to the change in credit risk since initial recognition, and for different stages, different loan loss provisioning rules apply.

Stage 1 consists of instruments for which there has been no considerable deterioration in credit risk since their initial recognition. In case there has been a significant worsening in the contract's credit risk (which not necessarily means delinquencies, but also considerable adverse changes in the macroeconomic outlook probably affecting the obligor), an instrument needs to be reclassified into Stage 2. In Stage 3, non-performing contracts reside. For Stage 1 loans (i.e. typically perfectly performing loans), a bank has to make loan loss provisions equal to the expected loss on the contract during the next *one-year* period. For contracts residing in Stage 2 or 3, provisions covering expected loan losses during their entire lifetime are necessary.

According to Abad and Suarez (2017), as a consequence of IFRS 9, a macroeconomic shock is going to increase banks' loan loss provisions through two channels. On the one hand, the deteriorating prospects immediately raise the impairment to be recognized for contracts with unchanged stage classification (increasing expected loss). On the other hand, as contracts are reclassified into worse stages, (in particular,

[†]The views expressed are those of the authors' and do not necessarily reflect the official view of the central bank of Hungary (Magyar Nemzeti Bank).

from Stage 1 into Stage 2), lifetime provisions must be made for an increasing number of loans. This could lead to substantial provisions immediately after the change in prospects, before even a single contract defaults. Thus, as shocks may have a different impact both in magnitude and dynamics than before, the rules of the new accounting standard have to be implemented in the top-down solvency stress tests of central banks in order to accurately map the connection between macroeconomic scenarios and banks' capital adequacy.

As in the new accounting framework, loan loss provisions have to be made for always performing loans as well, accounting for new loan originations has become more important. Besides, explicitly modeling both maturing and newly-disbursed loans is essential for stress testing in the current macroeconomic environment, in which banks' balance sheets are expanding quickly in many cases. Hence, the previously common static balance sheet assumption (in which we follow the banks' pre-existing loan portfolios during the stress testing horizon, i.e. maturing exposures and non-performing ones stay in the balance sheet, while there is no new loan origination) would underestimate the build-up of risks considerably. Therefore, stress tests have to put special emphasis on creating a versatile dynamic balance sheet framework, too.

Our credit risk stress testing model (incorporated into the Central Bank of Hungary's top-down macroprudential stress test as a satellite model) attempts to meet both requirements. In the literature, the closest approaches to our model are Skoglund and Chen (2017), who employ a quite similar modeling framework, and Abad and Suarez (2017), who rather emphasize the potential procyclical effects of the IFRS 9 standard. Our approach has the same basic structure, but it goes further in that we have estimated the transition probabilities and forecast loan loss provisions on real, contract-level data. Thus, our results represent the actual expected impact of the IFRS 9 standard on a real portfolio.

This article is structured as follows. Section 2 presents our framework for modeling loan loss provisions under IFRS 9, complemented with a dynamic balance sheet assumption. In Section 3 we discuss the main thoughts behind the estimation of transition probabilities, while Section 4 shows our results most interesting from a financial stability perspective. Section 5 concludes the summary.

2 The framework for forecasting loan loss provisions

For the purposes of stress testing, we have to give a prediction for the loan loss provisions and risk-weighted assets of each bank for *each period of the stress test's time horizon* conditional on the macroeconomic scenario. For the calculation of these predictions, we would like to take into account the requirements of IFRS 9 (provisions for prospective losses and stage-dependent provisioning rules).

The expected loan loss provision for a given instrument at a given future time consists of two parts: (1) the losses expected to be occurring up until that time, and (2) provisions for losses expected for the future at that given time. Moreover, the requirements for the latter differ based on the stage of the contract. For these reasons, we choose to model expected loan loss provisions in two steps:

- modeling the probability of each contract residing in a given stage at a future point in time, and
- *conditional on being in a given stage*, the expected losses for each contract on the corresponding time horizon.

The unconditional loan loss provisions (expected expected losses) can then be calculated using the law of total expectations.

We see two major avenues to calculating these complicated expectations. The first one is simulation on the contract level. The advantage of this approach is that it does not limit the choice of explanatory variables – in particular, history-dependent transition probabilities are admissible. On the other hand, its main drawback is its computational complexity due to the fact that expected losses conditional on each history would have to be calculated.

The other option is to impose strong restrictions on the transition process. In particular, if we assume that the processes determining the state of a contract have the Markov property (i.e. their development might depend on the state of the contract, but not on its history), and we model the process of states as an inhomogeneous Markov chain, analytical expressions for the above expectations can be derived. Most importantly, this assumption means that it cannot be taken into account (or only in a very limited way) in the modeling of the transition processes how long a contract has been in its current category or in which categories it has been in the past.[1] Under this assumption, the expected loan loss provisions can be calculated as

$$\begin{aligned}
\mathrm{E}\left(\mathrm{E}(L_t \mid k_t = k)\right) = & \sum_{k \in K_1} \mathrm{P}(k_t = k) \sum_{s=t+1}^{t+4} \{\mathrm{P}(k_s = D \cap k_{s-1} \neq D \mid k_t = k) \cdot LGD_s \cdot EAD_s\} \\
& + \sum_{k \in K_2} \mathrm{P}(k_t = k) \sum_{s=t+1}^{T} \{\mathrm{P}(k_s = D \cap k_{s-1} \neq D \mid k_t = k) \cdot LGD_s \cdot EAD_s\} \\
& + \sum_{k \in K_3} \mathrm{P}(k_t = k) \sum_{s=t_0}^{t} \{\underbrace{\mathrm{P}(k_s = D \cap k_{s-1} \neq D \mid k_t = k)}_{PD_{s|k_t=k}} \cdot LGD_s \cdot EAD_s\},
\end{aligned}$$

where k denotes the category in which the contract resides and D stands for the default category, and K_1, K_2 and K_3 are the sets of categories (possibly multiple per stage) corresponding to Stage 1, 2 and 3, respectively (contract indices are omitted for clarity). Losses in a given period are computed using the usual $PD \times LGD \times EAD$ factorization. In order for the above formula to be correct, the default category must be an absorbing state in the Markov process, otherwise multiple defaults of the same contract would entail charging the losses multiple times.[2]

With the Markov assumption, both the state probabilities $\mathrm{P}(k_t = k)$ and the conditional PDs $\mathrm{P}(k_s = D \cap k_{s-1} \neq D \mid k_t = k)$ can be easily expressed as functions of transition probabilities. Indeed, if we denote with y_t the vector of state probabilities at time t, then the probabilities of the contract residing at a time $u \geq t$ in the respective states may be calculated as:

$$y_u = y_t \prod_{h=t+1}^{u} \Gamma_h,$$

where Γ_h is the matrix of transition probabilities between the states of the model at time h. Furthermore, in the case $t + 1 = s$, the conditional PD above is just the element of Γ_{t+1} in the row and column corresponding to the initial state k and the default state D. If $s > t + 1$, we need to find the respective element of the product matrix $\prod_{h=t+1}^{s} \Gamma_h$.

In this framework, if we assume some (not path-dependent) processes for LGD_s and EAD_s, knowing the transition probabilities is sufficient to calculate the expected loan loss provisions analytically. Calculating expected risk-weighted assets is also relatively straightforward: if instead of assuming a process for EAD_s we more generally specify exposure processes for every state, in addition, we assume conditional risk weights to be constant, the unconditional RWA can be calculated using these and transition probabilities. We specify the exposure and LGD processes in Appendix A.1, while derive analytical formulas for the expected loan loss provision and the expected risk-weighted assets in Appendix A.2.

Due to the current economic outlook and the increased importance of new loan originations under IFRS 9, using a dynamic balance sheet for the forecast is fundamental. It implies that loan originations and terminations must also be modeled in addition to stage transitions. As a result of calculating exposures and loan loss provisions on the contract level, achieving this is conceptually simple and extremely flexible. New loans can be handled exactly the same way as already outstanding contracts, only with a later date of origination. The main difficulty lies in choosing assumptions for new loan originations, as all characteristics of each originated loan must be fully specified. To solve this, we chose a simple but relatively plausible assumption: banks disburse the same loans, at the same time of the year, to firms with the same characteristics as last year. This may then be further adjusted based on lending forecasts and expert opinions.

Terminating contracts is based on simpler rules. If a loan is performing at the time of maturity, it is removed from the balance sheet. Additionally, non-performing loans may also be removed from the balance sheet with a fixed, exogenously assumed probability in each period. In this case we assume no additional effect on earnings, but it affects the level of risk-weighted assets.

[1] History-dependence can be modeled by breaking a stage into multiple categories based on the possible paths of a variable. For example, Stage 1 of the standard is modeled as two separate categories. The first one consists of loans with no missed payments in the past, while the second one for loans currently performing, but problematic in the past. The use of this method is constrained by the rapid proliferation of categories and possible stage transitions.

[2] In fact, it is enough that the default stage only allows transitions to stages from which no further default is possible. To do this, we introduced a technical category, consisting of exposures not in the balance sheet any more.

Table 1: The states and possible transitions of the model

	(1)	(2)	(3)	(D)	(O)
Always performing (1)	R	–	1.	1.	–
Currently performing (2)	–	R	2.	2.	–
Delinquent for 30–90 days (3)	–	3.	3.	R	–
Delinquent for at least 90 days (D)	–	–	–	R	A
Out (O)	–	–	–	–	R

3 Modeling the transition processes

In this paper, we only detail the estimation of transition probabilities for the non-financial corporate loan portfolio of the Hungarian banking sector. For the estimation of these transition probabilities, we exploited the contract-level information of the Central Credit Information System (the credit register in Hungary), and connected it to enterprises' financial statement data from the National Tax and Customs Administration. We only included credit and loan contracts, loan-type framework agreements and financial leases in our sample. These three types represent the overwhelming majority of contracts in the credit register. We also discarded contracts of firms which have ever been categorized as financial firms during their observed histories, as the behavior of financial firms in terms of loans could fundamentally differ from that of non-financial companies. After these restrictions, we worked with around 1.5 million contracts and about 12.5 million observations, about 85 percent of the original data based on the number of contracts, and 89 percent based on the number of observations.

The time horizon of our estimation database starts at 2010 Q1, and has a quarterly frequency. Thanks to the availability of highly granular data, we were able to estimate transition probabilities at the contract level. We decided for the estimation at the contract level in spite of the fact that the obligor-level estimation would seem more logical as the loans of a company usually default at the same time, and the financial performance of a firm is probably more influenced by the parameters of its whole debt instead of its separate loans. The reason for this is that we need to forecast every explanatory variable of our transition models, and forecasting contract-level variables (which can approximate the features of a company's entire debt) is easier than forecasting individual corporate characteristics. Also, although the fact that loans of the same company have different PDs might seem strange at first, as we only want to calculate the expected losses on a portfolio, that is we only calculate a portfolio-level expected value, we are not interested in the interdependencies among the default of contracts.

There are five states in our model. The first state comprises contracts which have never been past due for more than 30 days so far. The second contains contracts that are currently performing, but have been in arrears previously. Contracts delinquent for 30 to 90 days are in the third, while contracts delinquent for more than 90 days are in the fourth, default category. Finally, contracts which have already been cleaned out of the balance sheet reside in the fifth, "out" state. Although both the first and second states belong to Stage 1 under IFRS 9, we modeled them separately as they have quite different risk characteristics. Furthermore, the definition of the third state does not involve the "significant deterioration in credit risk" criterion stipulated in IFRS 9, and thus does not exactly correspond to Stage 2.[3]

The logic of the transition matrix is depicted in Table 1. Fortunately, not every transition probability needs to be estimated, as some of them (marked by a dash) are not possible, and there is also one transition in each row the probability of which might be determined as a residual ("R"). Furthermore, the cleaning probability is based on an assumption ("A") instead of being modeled. The six transition probabilities left are estimated with three models. This is because some transition probabilities either have only a few actual occurrences to estimate on, and also because different transitions could represent essentially the same obligor behavior, thus there is no reason to estimate them separately.

Transition probabilities are modeled using a discrete-time proportional hazards model. In such models, the hazard rate of a transition is based on a (possibly time-dependent) baseline hazard, which is changed by the explanatory variables in a proportional manner. We assumed a constant baseline hazard,

[3]This is because we are calculating the PDs, our measure of credit risk, taking these states into consideration. Therefore, if we based our Stage 2 calculation on the PDs estimated this way, we would run into a circular reference: our calculated PDs would define the state of the contract, but transitions among states would change the PDs. Hence, we could not use the estimated PDs for the definition of the third state.

because the Markov property excludes it to be dependent on the time spent in the current category. Furthermore, we are observing an essentially continuous process at the ends of discrete intervals. As shown by Jenkins (2005, pp. 40-43), if an underlying process has the proportional hazard property, its discrete-time observations can be modeled as a generalized linear model using a complementary log-log link function.

For the estimation a set of firm-level variables, contract-level variables and macro variables were used. Estimating the effects of the latter on the transition probabilities is paramount as we are assessing the effect of an adverse macroeconomic scenario. A major constraint in the choice of explanatory variables is that due to the need to calculate lifetime expected losses, only variables that can be forecast up until the maturity of the longest contract can be chosen. This requirement practically excludes most balance sheet and P/L statement items, and only fixed or predictably changing characteristics may be used, such as size category, the year of origination, or the age of the company. Forecasts of macroeconomic variables are based on the macroeconomic forecasting model of the Central Bank of Hungary (detailed in Békési et al. (2016)). Following the end of the shock, the scenarios converge back to the macro model's steady state. The only contract-level explanatory variable we used that is not changing deterministically is the exchange rate depreciation since the time of loan origination. This is forecast assuming that all exchange rates related to the contracts in the sample will develop according to the macroeconomic forecast for the EUR/HUF exchange rate.

Table 2: Average partial effects of the main explanatory variables

	Size of shock	Size of impact (percentage points)		
		(1) ⇒ (3), (D)	(2) ⇒ (3), (D)	(3) ⇒ (2), (3)
Risk premium (t-1)	300 bps	0.38	-	-
Y-o-y difference of log real GDP (t-1)	-5 pps	0.38	-	-
Y-o-y difference of log end consumption (t-1)	-3 pps	-	6.83	-4.52
Exchange rate depreciation since loan origination (per cent) (t-1)	15 pps	0.06	-	-0.83
Average transition probability estimated for 2019 Q1 (per cent)	-	0.60	9.10	61.40

Note: The last row of the table contains the average transition probabilities estimated for 2019 Q1 using the baseline scenario, weighted by 2018 Q2 exposures. Bps: basis points, pps: percentage points.

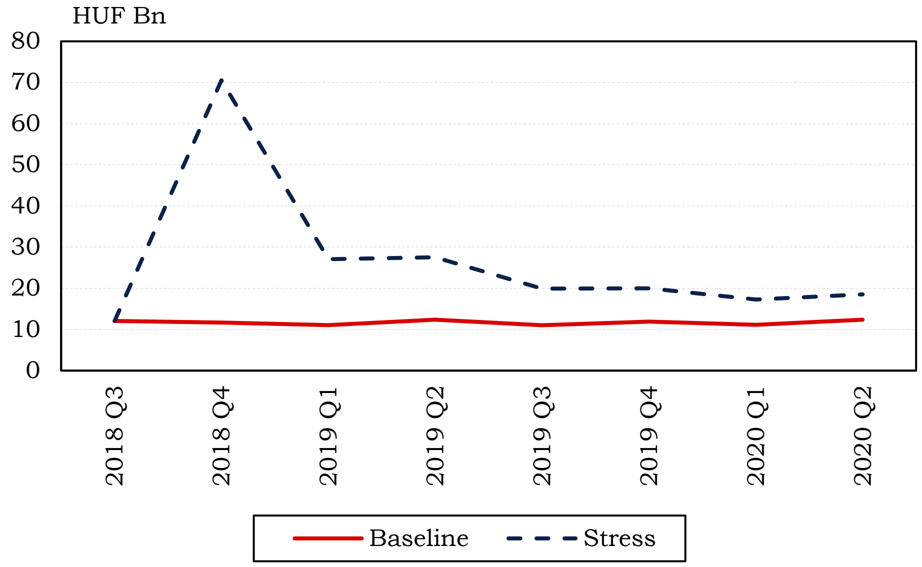
In all of the three models, the effects of the macro variables on transition probabilities have an intuitive sign, and are significant both statistically and economically. An increase in the risk premium or a slowdown of GDP-growth increases the probability of always performing loans transferring to Stages 2 and 3. A depreciation of the FX rate makes always performing loans becoming delinquent more likely, while decreasing the probability of problematic loans becoming performing again. Finally, a decrease in the growth rate of final consumption increases the chances of currently but not always performing loans becoming overdue again, and also decreases the probability of contracts resuming payments normally.

The estimation results of the three models of transition probabilities are detailed in section A.3 of the Appendix, and are shown in Table 3. Here, we would only like to emphasize the impact of the stress on predicted transition probabilities, which is reflected in the average partial effects of those explanatory variables in which the stress scenario materializes (Table 2). We examined the effects on transition probabilities not for an infinitesimal change in the macro variables, but for a shock-like change, for which we chose the shock sizes from earlier stress scenarios. To put the size of the impact in context, in the last row of the table we depicted the average transition probabilities for a period of the forecast horizon, estimated using the baseline scenario. Compared to these average transition probabilities, the sizes of the impact of shocks in explanatory variables seem sizeable, especially in the equations modeling the worsening of credit quality: assuming a plausibly-sized shock of a macro variable might modify the baseline transition probabilities by over 50 percent.

4 Results

With the model detailed above, for the stress testing horizon starting in 2018 Q3 and ending in 2020 Q2, we calculated loan loss provisions expected to be additionally allocated in each quarter, along both a baseline and a stress scenario[4], for the corporate portfolio of the Hungarian banking sector. Two aspects of the results are especially interesting from a financial stability perspective. First, as depicted in Figure 1, upon the realization of the shocks of the stress scenario in 2018 Q4, as a result of a shock-like change in expectations, a considerable amount of provisions must be allocated due to the increase of expected losses. Furthermore, due to transitions to worse categories, some additional impairment needs to be recognized throughout the time horizon. This development of loan loss provisions due to a shock is in stark contrast to impairment recognition under the previously applied accounting standards, in which impairments increased gradually as contracts defaulted. Therefore, the introduction of IFRS 9 loan loss provisioning will lead to a different impairment *volume*[5] *and dynamics* than the earlier approach, which might weaken banks' capital positions to larger degree than before, and might aggravate the procyclical behavior of the banking sector.

Figure 1: Loan loss provisions along the baseline and stress scenario under IFRS 9



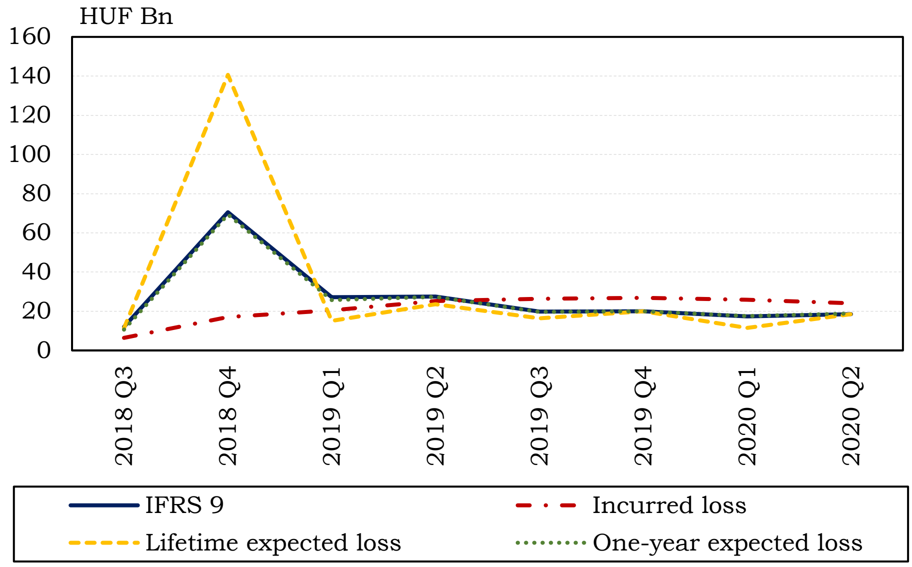
Note: For the non-financial corporate portfolio of the Hungarian banking sector, according to the baseline and stress scenarios of the 2018 November Financial Stability Report of the Central Bank of Hungary.

With our modeling framework we also calculated expected new loan loss provisions along the stress scenario under different provisioning requirements: the incurred loss approach, the lifetime and the one-year expected loss methods, as well as the combined logic of IFRS 9 (Figure 2). Although point-in-time in philosophy and not through-the-cycle, one-year expected losses are close to the calculations in the IRB approach for the capital requirement for credit risk. In turn, the lifetime expected loss approach is close to the one under implementation in the United States. The results show that when calculating lifetime expected losses for all states, about twice as much provisioning has to be recognized upon the occurrence of the shock than according to IFRS 9. Calculating one-year expected losses for all non-default

[4]These scenarios are the ones used in the solvency stress test of the Central Bank of Hungary's 2018 November Financial Stability Report.

[5]The amount of impairment expected to be recognized should be equal at the end of a contract's history regardless of using an accounting logic based on incurred or expected losses. Nevertheless, the volume of loan loss provisions for a bank will be higher than using the previous method as performing loans do not have incurred losses but do have expected ones.

Figure 2: Loan loss provisions along the stress scenario under various provisioning rules



Note: For the non-financial corporate portfolio of the Hungarian banking sector, according to the baseline and stress scenarios of the 2018 November Financial Stability Report of the Central Bank of Hungary.

states leads to roughly the same provisioning in the model as IFRS 9, as the only difference between the two provisioning rules is the category of 30 to 90 days of delinquency, which is not substantial in volume. Finally, using the incurred loss model, loan losses materialize only at defaults, which reinforces our previous conclusions.

5 Conclusion

The introduction of the IFRS 9 accounting standard is expected to change banks' loan loss provisioning dynamics to a considerable extent, which poses a challenge to stress testing frameworks. Additionally, due to the current macroeconomic outlook, the conventional static balance sheet assumption cannot be regarded as a conservative approach any more. As a response to these challenges, we have developed a credit risk modeling framework for top-down stress testing that (1) takes into account the new provisioning rules and (2) allows for a flexible specification of lending activity on the forecast horizon. We have utilized this framework for the non-financial corporate portfolio of the Hungarian banking system using transition probabilities estimated on its contract-level database.

Our results reinforce the presumption that the effect of an adverse macroeconomic shock on credit losses changes considerably both in terms of dynamics and magnitude as a result of the introduction of IFRS 9. In particular, the change in expectations has an immediate impact on loan loss provisions due to having to allocate impairment to cover increasing expected losses. Furthermore, due to transitions to worse categories, additional impairment needs to be recognized along the stress scenario. This is in contrast to impairment recognition under previous rules, and might exacerbate the procyclical behavior of the banking sector.

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A Appendix

A.1 Specification of exposure and LGD processes

As we would also like to calculate expected RWAs besides expected loan loss provisions, we not only need to specify the expected exposures at default, but also the expected exposures for each state. In our approach we assume that for non-default states, the expected exposure for a certain state and time does not depend on the history of the contract. For the default state, it was sufficient to our calculations to specify the EADs, i.e. the expected exposures at the time of defaulting, rather than those conditional on being in default. The expected exposures conditional on being in default are then the probability-weighted sums of EADs in possible previous default times. (This is only appropriate if we assume that the exposure does not change after the default, i.e. further instalments or interests for late payments are only taken into account implicitly, in the LGD.)

To empirically determine expected exposures for each state, we separated the so-called "base exposure", i.e. the exposure which would exist at a certain point in time in the case of no delinquencies, and the additional exposure stemming from the delinquencies represented in each state. To calculate base exposures, we classified the contracts according to repayment types, into loans with annuity-like repayment schemes, loans with repayment in full, exposures with linear capital repayment, and credit lines.

We computed base exposures according to the inherent logic of each group. For the group with annuity-like repayment, we used the contract's last observed exposure and its maturity to determine the annuity, in which the interest rate has been the banking-sector average interest rate of long-term coporate loans. The base exposure of contracts with repayment in full is given for all points in time of the forecast horizon by the last observed exposure. We calculated the base exposure of contracts with linear capital repayment using the last observation of the exposure and the contract's maturity. Finally, the expected exposure of credit lines was determined using the average drawdown rate of contracts in a certain state.

For states representing delinquencies, we assumed monthly instalments and determined the number of missed instalments based on the minimum number of days past due in a given state. We added these missed instalments to the base exposure, complemented with interest for late payments.[6]

For exposures denominated in foreign currency, we performed these calculations in that currency, and changed the resulting values to HUF using the exchange rate development of the macroeconomic scenario.

For the LGDs, we employed our expert judgement, fixing the LGD at 40 percent for the baseline, and at 50 percent for the stress scenario. This is a point in the model where further improvements would be necessary and are planned.

A.2 Derivation of the LLP and RWA for a point in time of the stress testing horizon

As we stated in the second section of the article, if we assume a macroeconomic scenario until the maturity of a given contract, as well as the contract's initial state, the expected loan loss provision and the expected

[6] We determined late payment interest rates as the average of those of larger banks, based on the general contract terms of their corporate loans.

risk-weighted assets can be derived knowing the transition matrix process and the LGD and exposure processes. Using the notations of the second section, this is shown in the following.

First, using the tower rule, the expected loan loss provision for a point in time t can be written as the expected value of loan loss provisions conditional on the state in time t:

$$\mathrm{E}(LLP_t) = \mathrm{E}\left(\mathrm{E}(LLP_t \mid k_t = k)\right).$$

Knowing that we have to calculate the loan loss provision differently for different stages, this may be written in a more detailed manner:

$$\begin{aligned}
\mathrm{E}\left(\mathrm{E}(LLP_t \mid k_t = k)\right) &= \sum_{k\in K} (\mathrm{E}(LLP_t \mid k_t = k) \cdot \mathrm{P}(k_t = k) \\
&= \sum_{k\in K_1} \mathrm{P}(k_t = k) \cdot \sum_{s=t+1}^{t+4} \{\mathrm{P}(k_s = D \cap k_{s-1} \neq D \mid k_t = k) \cdot LGD_s \cdot EAD_s\} \\
&+ \sum_{k\in K_2} \mathrm{P}(k_t = k) \cdot \sum_{s=t+1}^{T} \{\mathrm{P}(k_s = D \cap k_{s-1} \neq D \mid k_t = k) \cdot LGD_s \cdot EAD_s\} \\
&+ \sum_{k\in K_3} \mathrm{P}(k_t = k) \cdot \sum_{s=t_0}^{t} \{\mathrm{P}(k_s = D \cap k_{s-1} \neq D \mid k_t = k) \cdot LGD_s \cdot EAD_s\},
\end{aligned}$$

where in the case of states in K_1 and K_2, losses expected in the next year, and until the maturity T of the contract have to be recognized, respectively. In addition, in the case of the already nonperforming states in K_3, the expected loan loss provision is the expected value of losses already incurred until time t. The expected loss for a given period is the product of the probability of defaulting during that period, the LGD and the EAD pertaining to that period. Note that the probabilities $\mathrm{P}(k_t = k)$ and $\mathrm{P}(k_s = D \cap k_{s-1} \neq D \mid k_t = k)$ in the formula are the probabilities of staying in state k at time t, and defaulting in period s given staying in state k at time t, respectively. These are easy to calculate based on the transition matrix process. The only exceptions are the PDs conditional on being in the default or in the out states. For these, the condition pertaining to time t represents a terminal condition instead of an initial condition.

Assuming a deafult state in time t, we may calculate the PD in period s the following way:

$$\begin{aligned}
\mathrm{P}(k_s = D \cap k_{s-1} \neq D \mid k_t = D) =& \frac{\mathrm{P}(k_s = D \cap k_{s-1} \neq D \cap k_t = D)}{\mathrm{P}(k_t = D)} \\
=& \frac{\mathrm{P}(k_t = D \mid k_s = D \cap k_{s-1} \neq D) \cdot \mathrm{P}(k_s = D \cap k_{s-1} \neq D)}{\mathrm{P}(k_t = D)} \\
=& \frac{\mathrm{P}(k_t = D \mid k_s = D) \cdot \mathrm{P}(k_s = D \cap k_{s-1} \neq D)}{\mathrm{P}(k_t = D)}.
\end{aligned}$$

Here we used the definition of conditional probability twice, and the Markov assumption for the state process: knowing the state at time s, states at times earlier than s do not add additional information to the state probabilities for times later than s.

The second factor in the numerator of the resulting expression is an unconditional default probability, while the denominator is a state probability, which both are easy to calculate using the transition matrix process. For the first factor of the numerator:

$$\begin{aligned}
\mathrm{P}(k_t = D \mid k_s = D) =& \mathrm{P}(k_t = D \mid k_{s+1} = D \cap k_s = D) \cdot \mathrm{P}(k_{s+1} = D \mid k_s = D) \\
&+ \mathrm{P}(k_t = D \mid k_{s+1} \neq D \cap k_s = D) \cdot \mathrm{P}(k_{s+1} \neq D \mid k_s = D) \\
=& \mathrm{P}(k_t = D \mid k_{s+1} = D) \cdot \mathrm{P}(k_{s+1} = D \mid k_s = D).
\end{aligned}$$

In the first step, we only introduced another condition using the rule of total probability. The seemingly complicated expression becomes simpler if we employ the Markov assumption on the first part, and also notice that the first probability in the second part is zero, as from the default state a contract can only move to the out state, which is absorbing.

Therefore, we have iterated the condition in the original expression with one period. Iterating further, we obtain:

$$\mathrm{P}(k_t = D \mid k_s = D) = \prod_{j=s+1}^{t} \mathrm{P}(k_j = D \mid k_{j-1} = D).$$

The calculation is similar if we condition on the out state at time t.

To compute the expected value of the RWA, we assumed that banks follow the standard approach for the calculation of the capital requirement for credit risk. Thus, the RWA is given by the product of the risk weight and the net book value of the exposure:

$$\mathrm{E}(RWA_t) = \mathrm{E}\left(RW_t \cdot [EXP_t - LLP_t]\right) = \sum_{k \in K} \mathrm{E}\left(RW_t \cdot [EXP_t - LLP_t] \mid k_t = k\right) \cdot \mathrm{P}(k_t = k)$$

Although in the general case, the expected value of a product does not equal the product of the expectations, conditional on the state at time t, the risk weight is constant, and thus independent of EXP_t and LLP_t. Therefore, we may examine the expected values of the product one by one. As we have already calculated the conditional expected loan loss provision, we only need to focus on the conditional exposure $\mathrm{E}(EXP_t \mid k_t = k)$.

If we condition on non-default states, the exposure is assumed to be constant (Appendix A.1). Conditioning on the default state, we obtain the following expression:

$$\mathrm{E}(EXP_t \mid k_t = D) = \sum_{s=t_0}^{t} \mathrm{P}(k_s = D \cap k_{s-1} \neq D \mid k_t = D) \cdot \mathrm{E}(EXP_s \mid k_s = D \cap k_{s-1} \neq D \cap k_t = D).$$

The first factors have already been derived, while the second ones are the EADs. The calculation of the expression in which we condition on residing in the out state at time t follows in a similar way.

A.3 Detailed estimation results

The detailed results of the transition models introduced in section 3 are depicted in Table 3. The first column contains the estimation results for the probability of moving from the always performing state to the states with delinquencies of 30-90 days and of at least 90 days. In the second column, the estimates for the probability of transition from the currently perfoming state to the same delinquent states as in the first column are shown. Finally, the third column depicts the results of the model estimating the recovery from the state of a delinquency of 30-90 days to both the same state (indicating at least partial performance) and the currently performing category.

The effect of the macro variables has been described in section 3. The contract- and and corporate-level variables are mostly significant and of the intuitive sign in models estimating defaulting from performing states. In these models, the probability of delinquency and default is the smaller the larger the SME category[7], and the higher the category of the company's age. In addition, in the model explaining the transition from the always performing category to delinquency and default, a company's export activity[8] and its majority foreign ownership both significantly decrease its transition probability.

The coefficients estimated for the year of the loan origination are the result of two different effects. First, the coeficient of a year reflects the average riskiness of loans disbursed in that particular year. Besides, the earlier the origination of a loan took place, the longer was it able to avoid default, thus among the sample of loans extended earlier, there is a substantially smaller share of short-term loans or contracts that defaulted relatively early. In light of this, the fact that in our model the probability of transition from the always performing category decreased significantly in the years after 2008 reflects a considerable tightening of lending conditions after the crisis.

Apart from macro variables, we refrain from explaining the results of the model for the recovery from delinquency, as the effect of explanatory variables on the probability of recovery needs to be interpreted conditional on being in delinquency. That, however, is not necessarily intuitive: for example, although it

[7] We classified businesses as micro, small, medium-sized and large enterprises based on the threshold values for common European Union categories. Additionally, we categorized a firm as government-owned if at least 25 percent of it is owned by the (central or local) government.

[8] The dummy variable for export activity takes on one if the firm had any export activity in at least one of the years under review, and zero otherwise.

is less likely for a larger firm to default, if delinquency had already occurred, it is not straightforward whether a smaller or a larger firm would recover easier from default. Non-interpretability is not a problem, as contract- and firm-level variables have only been employed as controls.

Table 3: Estimation results of the transition models

	(1) ⇒ (3), (D)		(2) ⇒ (3), (D)		(3) ⇒ (2), (3)	
Macro variables						
Risk premium (t-1)	0.087	***	-		-	
Y-o-y difference of log real GDP (t-1)	-5.243	***	-		-	
Y-o-y difference of log end consumption (t-1)	-		-14.952	***	4.265	***
Exchange rate depreciation since loan origination (per cent) (t-1)	0.293	***	-		-0.155	***
Contract- or corporate-level variables						
SME category (reference: micro-enterprises)						
Small	-0.495	***	-0.217	***	0.035	***
Medium-sized	-0.613	***	-0.616	***	0.026	
Large	-1.098	***	-0.783	***	-0.493	***
(Partially) owned by central or local government	-0.983	***	-0.490	***	-0.021	
Majority foreign ownership	-0.174	***	-		0.008	
Export activity	-0.130	***	-0.013		0.150	***
Company age (reference: 1 closed year at most)						
2-5 years	-0.192	***	-0.210	***	-0.018	
6-10 years	-0.471	***	-0.449	***	0.038	
11-15 years	-0.713	***	-0.574	***	0.057	*
16-20 years	-0.887	***	-0.643	***	0.052	.
21-30 years	-1.039	***	-0.746	***	0.032	
Over 30 years	-2.389	***	-1.073	***	0.298	**
Year of loan origination (reference: 2008)						
2000 and before	-0.049		-0.966	**	-1.709	***
2001	-0.230	*	-0.439		-1.430	***
2002	-0.551	***	-0.370	*	-1.428	***
2003	-0.690	***	-0.618	***	-0.728	***
2004	-0.485	***	-0.242	**	-0.687	***
2005	-0.609	***	-0.139	***	-0.028	
2006	-0.240	***	-0.018		-0.002	
2007	-0.022		-0.023		-0.009	
2009	-0.107	***	-0.097	***	-0.060	***
2010	-0.167	***	-0.114	***	-0.086	***
2011	-0.184	***	-0.083	***	-0.168	***
2012	-0.327	***	0.060	*	-0.038	*
2013	-0.522	***	0.171	***	-0.032	
2014	-0.652	***	0.041		0.100	***
2015	-0.703	***	0.057		0.130	***
2016 and after	-0.656	***	0.046		0.167	***
Intercept	-3.819	***	-1.329	***	0.032	
Area under ROC curve	0.715		0.638		0.596	

Note: Estimated transitions are depicted using the notations of Table 1. Notation of significance levels: '***' – 0.1 percent, '**' – 1 percent, '*' – 5 percent, '.' – 10 percent. In the estimations, the region of the company's headquarters and the NACE economic sector of its main activity are also included as control variables, but their coefficients have been omitted from the table to save space.